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SCATTERING OF NONSPHERICAL PARTICLES REBOUNDING FROM A SMOOTH AND A ROUGH SURFACE IN A HIGH-SPEED GAS–PARTICLE FLOW

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Scattering indicatrices of nonspherical particles rebounding from a smooth and a rough surface are obtained by direct Monte Carlo simulations. Particles shaped as ellipsoids of revolution, rectangular prisms, and prisms with truncated vertices are considered. Surface roughness is defined as a two-dimensional profile whose scattering characteristics are close to those of real roughness induced by abrasive erosion of the surface in a high-speed gas–particle flow. Impact interaction of an individual particle with the surface is considered in a three-dimensional formulation. The scattering indicatrices of reflected particles are found to depend substantially on the particle shape in the case of rebound from a smooth surface and to be almost independent of the particle shape if the particles rebound from a rough surface.

Key words: *impact interaction of particles with the wall, nonspherical particles, smooth and rough surfaces, scattering indicatrices.*

Introduction. In a two-phase gas–particle flow over a body or an obstacle, the particles usually collide with the surface and rebound (reflect) from the latter. Particle reflection plays an important role in formation of the flow structure and the fields of parameters of the disperse phase. In most papers dealing with numerical simulations of confined gas–particle flows, the particles are assumed to be spherical and the surfaces are assumed to be smooth; in addition, various models of reflection of an individual particle from the wall are used. In real dusty gas flows, however, the particle shape normally differs from spherical (ashes, silica sand, and various commercial powders), which is responsible for the random character of their reflection. Such particles are scattered after rebounding from the surface, even if the surface is smooth. Real surfaces are normally rough owing to their prior mechanical treatment. Moreover, even an initially smooth surface rapidly becomes rough in a high-speed flow because of abrasive erosion. Surface roughness is the second important factor responsible for the random character of particle reflection.

It was noted [1–3] that it is important to take surface roughness into account, and various approaches to roughness modeling were proposed. Tsirkunov and Panfilov [4] reviewed and analyzed various models of surface roughness. Sommerfeld [5] demonstrated that the calculated and experimental parameters of motion of reflected particles can be substantially different because of the assumption of a spherical shape, even in the case of isometric particles. Crowe et al. [6] described the general approach to formulating and solving the problem of rebound of an arbitrarily shaped particle from a smooth surface, as applied to modeling of two-phase gas-particle flows. The increasing interest in effects of surface roughness and nonspherical particle shapes is caused by the development of more realistic numerical models of two-phase flows in the vicinity of confining surfaces.

The main objective of the present work was to study the characteristics of random scattering of nonspherical particles reflected from a smooth and a rough surface. Particles of three different shapes were considered: ellipsoids

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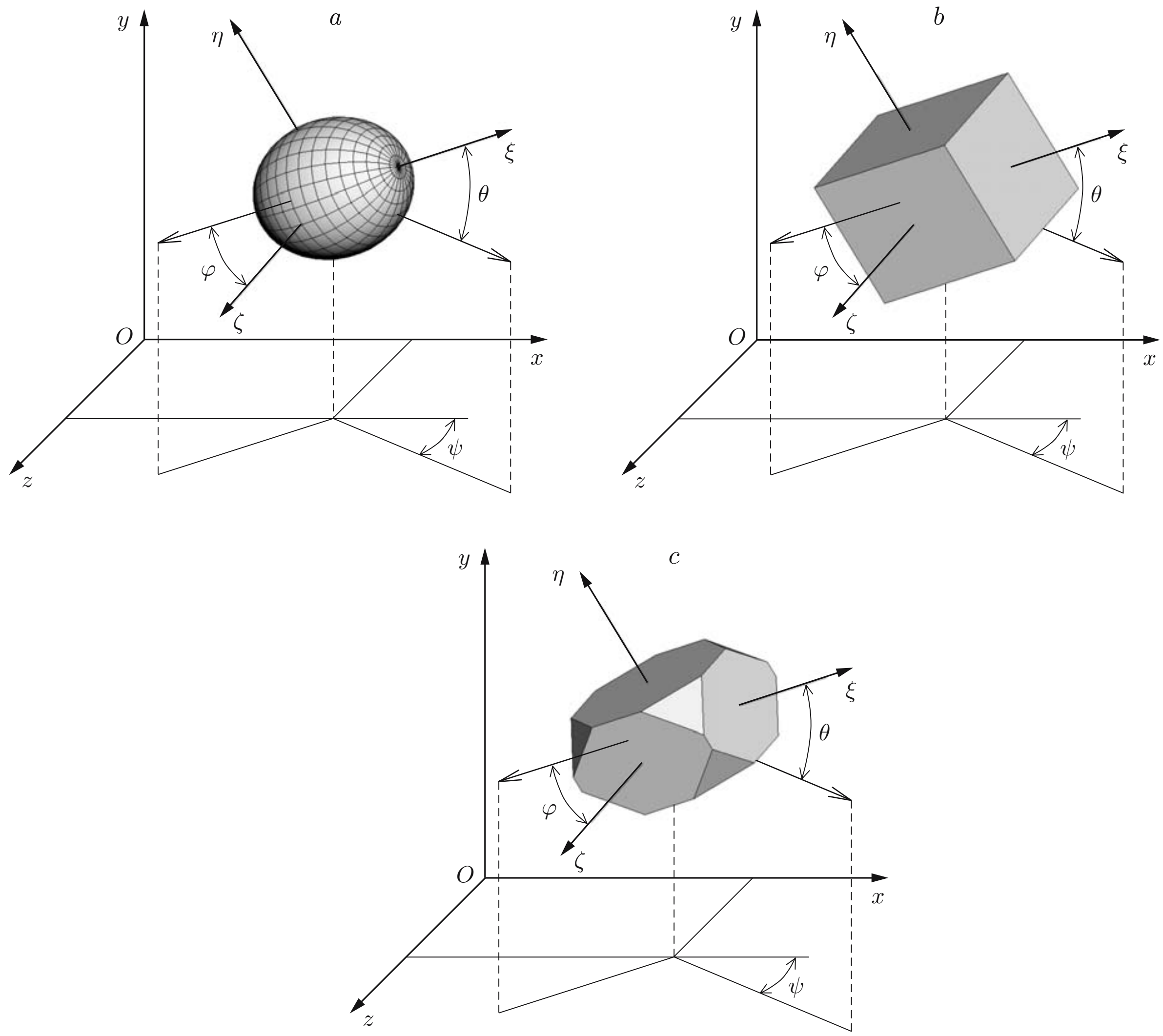

Fig. 1. Schematic orientation of particles of different shapes: ellipsoid (a), prism (b), and rectangular prism with truncated vertices (c).

of revolution, rectangular prisms, and prisms with truncated vertices. These shapes are close to the shapes of many real particles. Roughness caused by abrasive erosion of the surface exposed to a high-speed flow of a gas with solid particles was studied. The impact of particles on the wall was considered in a three-dimensional formulation. A similar problem in a simplified two-dimensional formulation was previously considered in [7].

1. Model of Impact Interaction of a Particle with a Solid Surface. First we analyze the particle interaction with a smooth flat wall. We introduce a right-hand Cartesian coordinate system $Oxyz$, such that the plane Oxz coincides with the wall surface, and the unit vector of the Oy axis is external with respect to the wall. The location of a nonspherical particle moving in physical space is determined not only by the coordinates of its center of mass (x_p, y_p, z_p) but also by the particle orientation. We also introduce a moving particle-fixed right-hand Cartesian coordinate system $O_p\xi\eta\zeta$, whose origin coincides with the center of mass of the particle, and the axes are directed along the main axes of inertia. The orientation of this particle-fixed coordinate system (and, hence, of the particle itself) with respect to the coordinate system $Oxyz$ at any instant of time is defined by three Euler angles (φ, ψ, and θ) (Fig. 1).

Let the velocity vector of the center of mass of the particle $\boldsymbol{V}_p$, the vector of angular velocity of particle rotation $\boldsymbol{\omega}_p$ before the impact, and particle orientation (angles φ, ψ, and θ) at the impact instant be known. The sought quantities are the velocities $\boldsymbol{V}_p$ and $\boldsymbol{\omega}_p$ after the impact. The time of impact interaction of the particle with

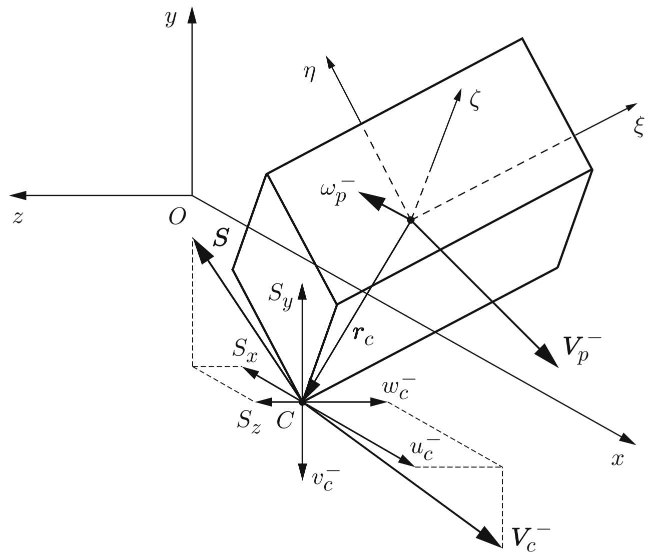

Fig. 2. Schematic interaction of a particle with a flat surface: C is the contact point.

the wall δt is assumed to be very small; hence, the impact can be considered as instantaneous. We also assume that the particle location with respect to the wall is not changed by the impact, and the particle–surface contact during the impact occurs at a point (Fig. 2). If the contact configuration is a line or a surface (i.e., if the particle collides with the surface by the prism rib or face), the contact point is assumed to be the geometric center of the corresponding line or surface.

We write the equations that describe the changes in the momentum and the angular momentum of the particle during the impact with the particle parameters before and after the impact denoted by the superscripts minus and plus:

$$\begin{gathered} m_p \Delta \boldsymbol{V}_p \equiv m_p(\boldsymbol{V}_p^+ - \boldsymbol{V}_p^-) = \int\limits_0^{\delta t} \boldsymbol{f}_c(t)\, dt = \boldsymbol{S}, \\ \|J_p\|\, \Delta \boldsymbol{\omega}_p \equiv \|J_p\|\, (\boldsymbol{\omega}_p^+ - \boldsymbol{\omega}_p^-) = \int\limits_0^{\delta t} [\boldsymbol{r}_c \times \boldsymbol{f}_c(t)]\, dt = \boldsymbol{r}_c \times \boldsymbol{S}. \end{gathered} \tag{1}$$

Here m_p is the particle mass, $\boldsymbol{f}_c$ and $\boldsymbol{S}$ are the force and the impact momentum acting on the particle at the contact point, $\|J_p\|$ is the tensor of inertia of the particle, and $\boldsymbol{r}_c$ is the radius vector determining the location of the contact point with respect to the center of mass of the particle (see Fig. 2).

We also introduce the velocity of the contact point of the particle $\boldsymbol{V}_c$. The change in this velocity due to the impact is described by the kinematic relation

$$\Delta \boldsymbol{V}_c \equiv \boldsymbol{V}_c^+ - \boldsymbol{V}_c^- = \Delta \boldsymbol{V}_p + \Delta \boldsymbol{\omega}_p \times \boldsymbol{r}_c. \tag{2}$$

It follows that $\Delta \boldsymbol{V}_p = \Delta \boldsymbol{V}_c - \Delta \boldsymbol{\omega}_p \times \boldsymbol{r}_c$. Substituting this expression into the left side of the first equation in system (1), we find $\boldsymbol{S}$. Then, substituting the result into the second equation, we obtain

$$m_p^{-1} \|J_p\|\, \Delta \boldsymbol{\omega}_p = \boldsymbol{r}_c \times \Delta \boldsymbol{V}_c - \boldsymbol{r}_c \times [\Delta \boldsymbol{\omega}_p \times \boldsymbol{r}_c]. \tag{3}$$

The vector relation (3) contains two unknown vectors $\Delta \boldsymbol{V}_c$ and $\Delta \boldsymbol{\omega}_p$. The vector $\Delta \boldsymbol{V}_c$ can be defined by setting the coefficients of recovery of the normal and tangent-to-the-wall components of the particle velocity vector at the contact point: $a_{nc} = -V_{cn}^+/V_{cn}^-$ and $a_{\tau c} = V_{c\tau}^+/V_{c\tau}^-$. In the present work, we assume that the coefficient $a_{\tau c}$ equals zero (this condition corresponds to the absence of particle slipping at the contact point at the moment of particle rebound from the surface) and $a_{nc} = 0.8$, which is a certain mean value for the impact angles α_1 (see below) ranging from 0 to $\approx 40°$ in the range of impact velocities from 50 to 350 m/sec [8].

Let u_p, v_p, and w_p and u_c, v_c, and w_c be the components of the vectors $\boldsymbol{V}_p$ and $\boldsymbol{V}_c$, respectively, in the coordinate system $Oxyz$. Then, the components Δu_c, Δv_c, and Δw_c of the vector $\Delta \boldsymbol{V}_c$ are expressed via the components of the vector $\boldsymbol{V}_c$ and the coefficients a_{nc} and $a_{\tau c}$ as follows:

$$\Delta u_c = -u_c^-, \qquad \Delta v_c = -(a_{nc}+1)v_c^-, \qquad \Delta w_c = -w_c^-. \tag{4}$$

It seems reasonable to consider the rotational motion of the particle and the components of its angular velocity in the particle-fixed coordinate system $O_p\xi\eta\zeta$, because the tensor $\|J_p\|$ in this case has only diagonal nonzero components $J_{p\xi}$, $J_{p\eta}$, and $J_{p\zeta}$, which are the main moments of inertia of the particle. We write the vector relation (3) in projections onto the axes of the particle-fixed coordinate system:

$$\begin{aligned}
\hat{J}_{p\xi}\Delta\omega_{p\xi} &= \eta_c\Delta V_{c\zeta} - \zeta_c\Delta V_{c\eta} - \eta_c^2\Delta\omega_{p\xi} + \xi_c\eta_c\Delta\omega_{p\eta} + \xi_c\zeta_c\Delta\omega_{p\zeta} - \zeta_c^2\Delta\omega_{p\xi},\\
\hat{J}_{p\eta}\Delta\omega_{p\eta} &= \zeta_c\Delta V_{c\xi} - \xi_c\Delta V_{c\zeta} - \zeta_c^2\Delta\omega_{p\eta} + \eta_c\zeta_c\Delta\omega_{p\zeta} + \eta_c\xi_c\Delta\omega_{p\xi} - \xi_c^2\Delta\omega_{p\eta},\\
\hat{J}_{p\zeta}\Delta\omega_{p\zeta} &= \xi_c\Delta V_{c\eta} - \eta_c\Delta V_{c\xi} - \xi_c^2\Delta\omega_{p\zeta} + \zeta_c\xi_c\Delta\omega_{p\xi} + \zeta_c\eta_c\Delta\omega_{p\eta} - \eta_c^2\Delta\omega_{p\zeta}.
\end{aligned} \tag{5}$$

Here $\hat{J}_{pi} = J_{pi}/m_p$ $(i \equiv \xi, \eta, \zeta)$ and ξ_c, η_c, and ζ_c are the components of the radius vector $\boldsymbol{r}_c$. Relations (5) are a system of linear algebraic equations with respect to $\Delta\omega_{p\xi}$, $\Delta\omega_{p\eta}$, and $\Delta\omega_{p\zeta}$, which has the following matrix form:

$$\begin{pmatrix} \hat{J}_{p\xi}+\eta_c^2+\zeta_c^2 & -\xi_c\eta_c & -\xi_c\zeta_c \\ -\eta_c\xi_c & \hat{J}_{p\eta}+\zeta_c^2+\xi_c^2 & -\eta_c\zeta_c \\ -\zeta_c\xi_c & -\zeta_c\eta_c & \hat{J}_{p\zeta}+\xi_c^2+\eta_c^2 \end{pmatrix} \begin{pmatrix} \Delta\omega_{p\xi} \\ \Delta\omega_{p\eta} \\ \Delta\omega_{p\zeta} \end{pmatrix} = \begin{pmatrix} \eta_c\Delta V_{c\zeta} - \zeta_c\Delta V_{c\eta} \\ \zeta_c\Delta V_{c\xi} - \xi_c\Delta V_{c\zeta} \\ \xi_c\Delta V_{c\eta} - \eta_c\Delta V_{c\xi} \end{pmatrix}. \tag{6}$$

By solving this system, we obtain the values of $\Delta\omega_{p\xi}$, $\Delta\omega_{p\eta}$, and $\Delta\omega_{p\zeta}$. The components of the vector of angular velocity of the particle at the rebound moment can be easily obtained in the particle-fixed coordinate system

$$\omega_{p\xi}^+ = \omega_{p\xi}^- + \Delta\omega_{p\xi}, \qquad \omega_{p\eta}^+ = \omega_{p\eta}^- + \Delta\omega_{p\eta}, \qquad \omega_{p\zeta}^+ = \omega_{p\zeta}^- + \Delta\omega_{p\zeta}. \tag{7}$$

To study the scattering of reflected particles, we have to determine the rebound direction of an individual particle with an arbitrary orientation prior to the impact with the surface. Obviously, this direction coincides with the direction of the velocity vector of the center of mass of the particle at the rebound moment $\boldsymbol{V}_p^+$. Taking into account relation (2), we can calculate the vector $\boldsymbol{V}_p^+$ as follows:

$$\boldsymbol{V}_p^+ = \boldsymbol{V}_p^- + \Delta\boldsymbol{V}_p = \boldsymbol{V}_p^- + \Delta\boldsymbol{V}_c - \Delta\boldsymbol{\omega}_p \times \boldsymbol{r}_c. \tag{8}$$

The velocity vector of the center of mass of the particle before the impact $\boldsymbol{V}_p^-$ is usually defined (or is obtained from particle trajectory calculations) by the components u_p^-, v_p^-, and w_p^- in the stationary coordinate system $Oxyz$. The components of the vector $\Delta\boldsymbol{V}_c$ in the right side of Eq. (8) are determined in this coordinate system by relations (4), where u_c^-, v_c^-, and w_c^- can be found from the kinematic relation $\boldsymbol{V}_c = \boldsymbol{V}_p + \boldsymbol{\omega}_p \times \boldsymbol{r}_c$. Thus, to find the components u_p^+, v_p^+, and w_p^+ of the vector $\boldsymbol{V}_p^+$ with the use of Eq. (8), we have to calculate the vectors $\boldsymbol{\omega}_p$, $\Delta\boldsymbol{\omega}_p$, and $\boldsymbol{r}_c$ in the coordinate system $Oxyz$, which are found from the equations of particle dynamics, usually in the particle-fixed coordinate system $O_p\xi\eta\zeta$.

In the coordinate systems $Oxyz$ and $O_p\xi\eta\zeta$, the components of any vector $\boldsymbol{b}$ are related as [9]

$$\begin{pmatrix} b_\xi \\ b_\eta \\ b_\zeta \end{pmatrix} = A \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix}, \qquad \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = A^{\mathrm{t}} \begin{pmatrix} b_\xi \\ b_\eta \\ b_\zeta \end{pmatrix}, \tag{9}$$

where A is the rotation transformation matrix whose elements can be expressed via the Euler angles φ, ψ, and θ [9]; A^{t} is the transposed matrix (A^{t} coincides with the inverse matrix A^{-1}). In the description of kinematics of rotational motion of the particle, however, it seems reasonable to use the mathematical apparatus of Rodrigues–Hamilton quaternions rather than the Euler angles, because the system of kinematic equations has no singularities in such a case [10]. Formally, this means the transition to new variables λ_k $(k = 0, 1, 2, 3)$ related to the angles φ, ψ, and θ as

$$\begin{aligned}
\lambda_0 &= \cos(\varphi/2)\cos(\psi/2)\cos(\theta/2) - \sin(\varphi/2)\sin(\psi/2)\sin(\theta/2),\\
\lambda_1 &= \sin(\varphi/2)\cos(\psi/2)\cos(\theta/2) + \cos(\varphi/2)\sin(\psi/2)\sin(\theta/2),\\
\lambda_2 &= \cos(\varphi/2)\sin(\psi/2)\cos(\theta/2) + \sin(\varphi/2)\cos(\psi/2)\sin(\theta/2),\\
\lambda_3 &= \cos(\varphi/2)\cos(\psi/2)\sin(\theta/2) - \sin(\varphi/2)\sin(\psi/2)\cos(\theta/2).
\end{aligned}$$

The matrix A is expressed through the variables λ_k as follows:

$$A = \begin{bmatrix} \lambda_0^2+\lambda_1^2-\lambda_2^2-\lambda_3^2 & 2(\lambda_0\lambda_3+\lambda_1\lambda_2) & 2(\lambda_1\lambda_3-\lambda_0\lambda_2) \\ 2(\lambda_1\lambda_2-\lambda_0\lambda_3) & \lambda_0^2-\lambda_1^2+\lambda_2^2-\lambda_3^2 & 2(\lambda_0\lambda_1+\lambda_2\lambda_3) \\ 2(\lambda_0\lambda_2+\lambda_3\lambda_1) & 2(\lambda_2\lambda_3-\lambda_0\lambda_1) & \lambda_0^2-\lambda_1^2-\lambda_2^2+\lambda_3^2 \end{bmatrix}. \tag{10}$$

Knowing the angles φ, ψ, and θ responsible for the spatial orientation of the particle at the impact moment or, which is the same, λ_0, λ_1, λ_2, and λ_3, we calculate the elements of the matrix A and find, using the second relation in (9),

$$\begin{pmatrix} \omega_{px} \\ \omega_{py} \\ \omega_{pz} \end{pmatrix} = A^{\mathrm{t}} \begin{pmatrix} \omega_{p\xi} \\ \omega_{p\eta} \\ \omega_{p\zeta} \end{pmatrix}, \quad \begin{pmatrix} \Delta\omega_{px} \\ \Delta\omega_{py} \\ \Delta\omega_{pz} \end{pmatrix} = A^{\mathrm{t}} \begin{pmatrix} \Delta\omega_{p\xi} \\ \Delta\omega_{p\eta} \\ \Delta\omega_{p\zeta} \end{pmatrix}, \quad \begin{pmatrix} r_{cx} \\ r_{cy} \\ r_{cz} \end{pmatrix} = A^{\mathrm{t}} \begin{pmatrix} \xi_c \\ \eta_c \\ \zeta_c \end{pmatrix}. \tag{11}$$

After these transformations, we use Eq. (8) to obtain the final expressions for the components of the vector $\boldsymbol{V}_p^+$ in the coordinate system $Oxyz$:

$$\begin{aligned} u_p^+ &= u_p^- + \Delta u_c - \Delta\omega_{py} r_{cz} + \Delta\omega_{pz} r_{cy}, \\ v_p^+ &= v_p^- + \Delta v_c - \Delta\omega_{pz} r_{cx} + \Delta\omega_{px} r_{cz}, \\ w_p^+ &= w_p^- + \Delta w_c - \Delta\omega_{px} r_{cy} + \Delta\omega_{py} r_{cx}. \end{aligned} \tag{12}$$

Thus, if the velocity of the center of mass of the particle $\boldsymbol{V}_p^-$, its angular velocity $\boldsymbol{\omega}_p^-$, and particle orientation in space (values of λ_0, λ_1, λ_2, and λ_3) are known at the impact moment, we can calculate the velocity of the center of mass of the particle and its angular velocity at the rebound moment in the coordinate system $Oxyz$ using the kinematic dependence $\boldsymbol{V}_c^- = \boldsymbol{V}_p^- + \boldsymbol{\omega}_p^- \times \boldsymbol{r}_c$ and relations (4), (6), (7), and (10)–(12). Note that the location of the contact point with respect to the center of mass $\boldsymbol{r}_c$ is uniquely determined by particle orientation at the impact moment.

Thus, we analyzed particle reflection from a smooth surface. To apply the same approach for determining particle parameters in the case of its rebound from a rough surface, we need to specify the surface relief. (Various aspects of modeling of this relief are considered below.) In the present paper, we consider particles whose diameter is smaller than the mean distance between the neighboring relief peaks (such a situation is typical of roughness induced by abrasive erosion). Therefore, the incident particle reflected from the wall can collide with the surface of the rough relief several times (Fig. 3). The following scheme is used to calculate particle interaction with a rough relief. The problem of the first impact is considered similarly to the problem of particle impact on a smooth flat surface, but the coordinate system $Oxyz$ is introduced in a plane tangential to the relief surface at the particle–surface contact point. After particle rebound, we calculate the inertial motion of the freely rotating particle and ignore the forces and torque acting from the carrier gas. If the particle collides with the relief for the second time, we again solve the problem of its impact on the surface (construct a plane tangential to the relief surface at the new contact point, etc.). The particle is assumed to be finally and completely reflected if it leaves the region of the relief. The parameters of particle motion after the last collision with the relief are taken as the parameters of particle reflection from the surface. We denote the velocity vector of the center of mass of the particle and the vector of its angular velocity by $\boldsymbol{V}_{p1}$ and $\boldsymbol{\omega}_{p1}$ before the first collision with the surface and by $\boldsymbol{V}_{p2}$ and $\boldsymbol{\omega}_{p2}$ after the last collision (see Fig. 3).

2. Model of the Rough Surface. The study of the surface of ductile metal samples subjected to a high-speed (100–300 m/sec) flow of a gas with solid particles showed that the roughness resulting from abrasive erosion for angles $\alpha_1 < 40^\circ$ (see Fig. 3) has the form of transverse waves whose profile depends on the particle size, particle velocity, and impact angle. One of these samples is shown in Fig. 4. The roughness structure has an essentially two-dimensional character. The roughness relief, therefore, was described in the present study by a two-dimensional profile $Y = Y_w(X)$ in the coordinate system OXY, where the axis OX is directed along the surface across the roughness "waves," and the axis OY is directed normal to the surface. By summarizing the measurements of the real roughness profiles on various samples by a "Rank Taylor Hobson" profilometer, we determined the dependence $Y = Y_w(X)$ (Fig 4c), which was very close to a quasi-periodic function with a certain random scattering of the period and amplitude.

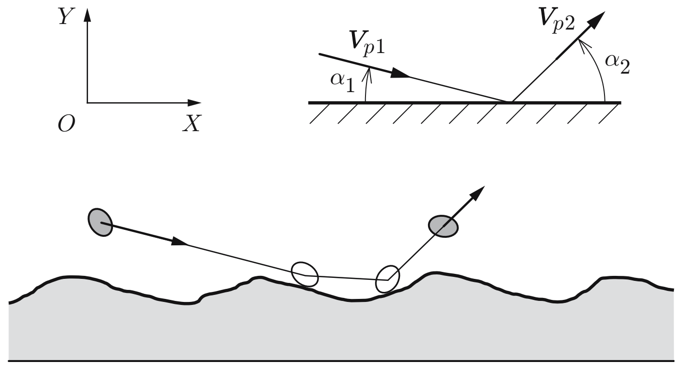

Fig. 3. Schematic interaction of a particle with a rough surface.

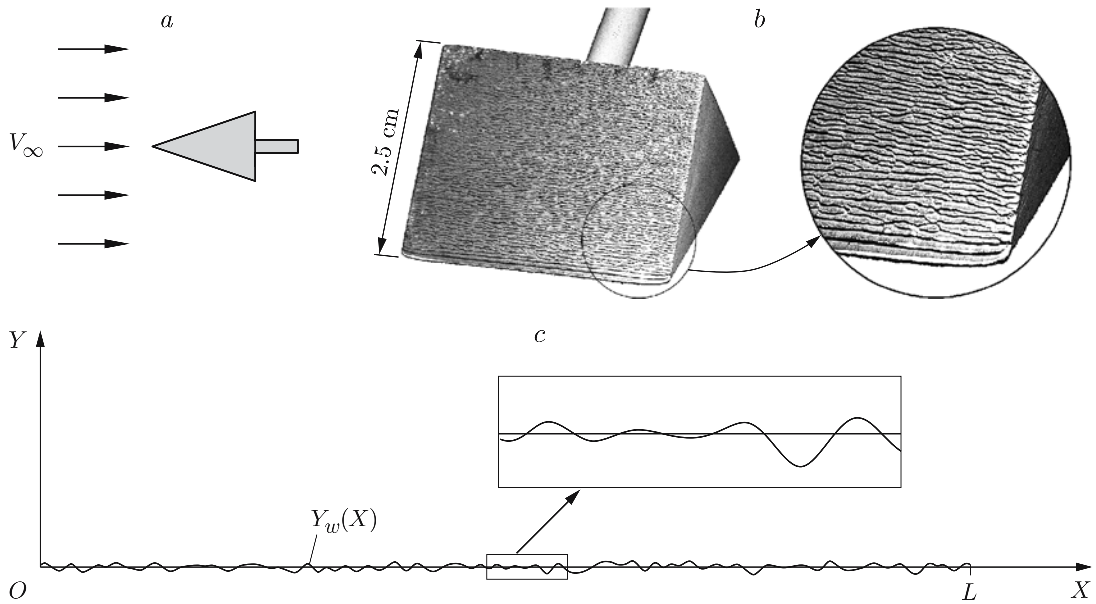

Fig. 4. Schematic of the flow (a), sample of the rough surface induced by abrasive erosion (b), and two-dimensional profile of roughness (c).

As the length of the samples of real profiles was comparatively small (about 2.5 cm), a numerically generated (model) profile of a greater length L was used to study the statistical characteristics of scattering of reflected particles. The value of L was chosen such that the scattering characteristics were almost independent of it. The following algorithm was used to construct the model profile. A sequence of K points was set in the plane OXY. The coordinates of these points were determined by the relations $X_i = X_{i-1} + \chi$ $(i = 2, \ldots, K)$ and $Y_i = \gamma$ $(i = 1, \ldots, K)$, where χ and γ are random quantities, which obey the normal distribution law with the mean values and standard deviations $M_\chi = h/2$, $\sigma_\chi < h/6$ and $M_\gamma = 0$, $\sigma_\gamma \leqslant Y_{w,\max}/3$, respectively. The parameters h and $Y_{w,\max}$ correspond to the mean step and the maximum height of roughness peaks. In generating the values of X_i and Y_i, if at least one of the values was outside the intervals $X_{i-1} + M_\chi \pm 3\sigma_\chi$ and $\pm 3\sigma_\gamma$, respectively, this value was rejected and generated again. The sequence of the resultant points (X_i, Y_i) was smoothed by a cubic spline, which was considered as the roughness profile $Y = Y_w(X)$. The parameters M_χ, σ_χ, and σ_γ were chosen under the conditions of the best fit of the scattering properties of the model and real profiles of roughness. In the present work, we used the values of the parameters $M_\chi = 80$ μm, $\sigma_\chi = 10$ μm, and $\sigma_\gamma = 20$ μm, which correspond to the scattering properties of the sample shown in Fig. 4.

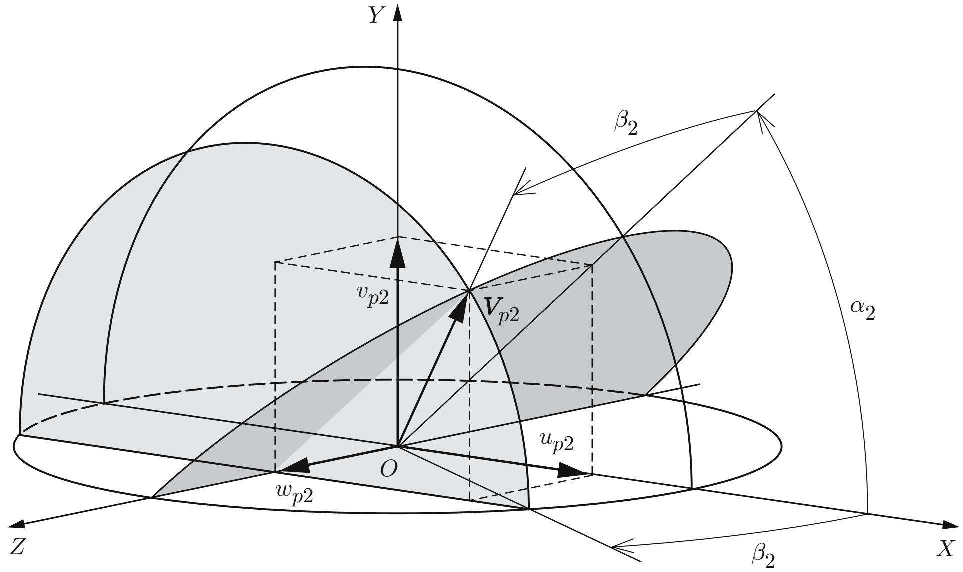

Fig. 5. Angles α_2 and β_2 determining the direction of particle reflection (orientation of the vector $\boldsymbol{V}_{p2}$) with respect to the coordinate system $OXYZ$.

3. Scattering of Reflected Particles. The particles incident onto a flat rough surface at a specified angle α_1 (see Fig. 3) with identical translational and angular velocities ($\boldsymbol{V}_{p1}$ and $\boldsymbol{\omega}_{p1}$) are reflected from the surface with different velocities $\boldsymbol{V}_{p2}$ (in magnitude and direction) because of the random orientations of the particles before the impact and their random locations with respect to the roughness relief. The phenomenon of rebound of particles with identical values of $\boldsymbol{V}_{p1}$, $\boldsymbol{\omega}_{p1}$, and α_1 in random directions is called the scattering of particles during their reflection from the surface. The direction of particle rebound can be described by the angles α_2 and β_2 (Fig. 5). The angle α_2 can vary from 0 to π, and the angle β_2 can vary from $-\pi/2$ to $\pi/2$. Let N be the number of particles with fixed parameters $\boldsymbol{V}_{p1}$ and $\boldsymbol{\omega}_{p1}$ incident onto the surface at an angle α_1 and $dN(\alpha_2, \beta_2, d\alpha_2, d\beta_2)$ be the number of particles reflected in the direction defined by the intervals of the angles $[\alpha_2, \alpha_2 + d\alpha_2]$ and $[\beta_2, \beta_2 + d\beta_2]$. We introduce the function $I(\alpha_2, \beta_2)$ of the distribution of the reflected particles with respect to the angles α_2 and β_2 by the relation $I(\alpha_2, \beta_2)\, d\alpha_2\, d\beta_2 = dN(\alpha_2, \beta_2, d\alpha_2, d\beta_2)/N$. The expression $I(\alpha_2, \beta_2)\, d\alpha_2\, d\beta_2$ is actually the probability of particle rebound in the direction (α_2, β_2) in the intervals of the angles $d\alpha_2$ and $d\beta_2$. The function $I(\alpha_2, \beta_2)$ will be called the spatial scattering indicatrix. Integrating $I(\alpha_2, \beta_2)$ with respect to β_2 from $-\pi/2$ to $\pi/2$, we obtain the scattering indicatrix in the plane OXY, which yields the distribution of reflected particles with respect to the angle α_2 only. We denote this indicatrix by $F(\alpha_2)$.

In this work, we found the scattering indicatrices of particles reflected from a smooth surface and from a rough surface by direct Monte Carlo simulations. A uniform rectangular grid with steps $\Delta\alpha_2 = \Delta\beta_2 = \pi/180$ was introduced in the domain of the variables $0 \leqslant \alpha_2 \leqslant \pi$ and $-\pi/2 \leqslant \beta_2 \leqslant \pi/2$. Reflection of a large number of particles ($N \approx 10^7$) was calculated; before the impact, the Euler angles φ, ψ, and θ for each particle were set in a random manner on the basis of a uniform distribution. For the rough surface, the particle location with respect to the roughness profile was randomly specified. We determined the number of reflected particles N_{ij} located in the grid cell (ij), i.e., particles whose reflection angles α_2 and β_2 were in the intervals $(i-1)\Delta\alpha_2 \leqslant \alpha_2 < i\,\Delta\alpha_2$ and $(j-1)\Delta\beta_2 \leqslant \beta_2 < j\,\Delta\beta_2$. Then, we calculated the ratio N_{ij}/N, which is close to the probability of particle reflection in the direction determined by these intervals of angles, if the value of N is sufficiently large. After that, we calculated an approximate value of the function I in the cell considered:

$$I(\alpha_2, \beta_2)_{ij} \approx \frac{N_{ij}}{N\Delta\alpha_2\Delta\beta_2}.$$

After calculating the values of $I(\alpha_2, \beta_2)_{ij}$ in all cells, we obtained the distribution function of reflected particles in the entire range of variation of the angles. The particle distribution with respect to the angle α_2 only [scattering

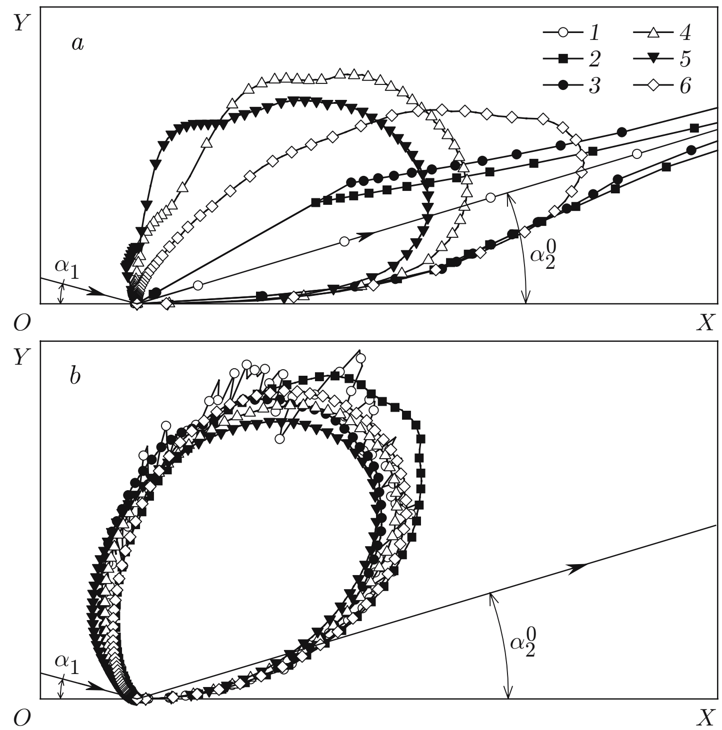

Fig. 6. Scattering indicatrices $F(\alpha_2)$ of reflected particles of different shapes from a smooth surface (a) and a rough surface (b): 1) sphere; 2) extended ellipsoid of revolution ($b/a = 0.8$); 3) flattened ellipsoid of revolution ($b/a = 1.25$); 4) extended prism ($b/a = c/a = 0.8$); 5) flattened prism ($b/a = c/a = 1.25$); 6) prism with truncated vertices ($b/a = 0.6$ and $c/a = 0.8$).

indicatrix $F(\alpha_2)$ in the plane OXY] was obtained by calculating the values of

$$F(\alpha_2)_i \approx \frac{1}{N\Delta\alpha_2} \sum_j N_{ij}$$

in all grid intervals along the axis α_2 [summation of the number of reflected particles in the last relation is performed over all cells (ij) with a fixed value of i].

Based on results of numerical simulations of scattering of differently shaped particles, we determined both the spatial indicatrices $I(\alpha_2, \beta_2)$ and the indicatrices $F(\alpha_2)$. The coordinate system was chosen such that the velocity vector $\boldsymbol{V}_{p1}$ lied in the plane OXY. The roughness was assumed to be two-dimensional and to be described by the profile in the plane OXY (see Sec. 2). We assumed that the particles did not rotate before their first impact onto the surface ($\boldsymbol{\omega}_{p1} = 0$). The particle shapes considered were extended ellipsoids of revolution with the axes ratio $b/a = 0.8$, flattened ellipsoids of revolution with the axes ratio $b/a = 1.25$, extended rectangular prisms with the ratio of the sides $b/a = c/a = 0.8$, flattened prisms with the ratio of the sides $b/a = c/a = 1.25$, and prisms with truncated vertices with the ratios of the sides $b/a = 0.6$ and $c/a = 0.8$ (see Fig. 1). The value of a was taken to be 32 μm. The calculations showed that variations of a in a wide range (provided that the value of a is smaller than the distance between the peaks of the roughness profile) has almost no effect on the form of the indicatrices.

As it could be expected, the spatial indicatrices turned out to be symmetric with respect to the plane OXY. We present the results for the indicatrices $F(\alpha_2)$ obtained for $\alpha_1 = 15°$. Figure 6a shows the scattering indicatrices of differently shaped particles reflected from a smooth wall. The indicatrices for ellipsoidal and prismatic particles are seen to be substantially different. Prismatic particles are much more scattered over the angle α_2, and the dominating angle of their rebound is substantially different from the angle of rebound of spherical particles α_2^0. In the case of particle reflection from a rough surface, the scattering indicatrices are considerably different (Fig. 6b). The indicatrices of particles of all shapes considered (and also spherical particles) are rather close to each other, and the most probable angle of particle rebound in the plane OXY is substantially greater than the most probable angle α_2 in the case of particle reflection from a smooth surface. The scatter of the values of the angle α_2 is also greater. A comparison of indicatrices in Figs. 6a and 6b shows that surface roughness induced by abrasive erosion leads to substantial differences in the characteristics of scattering of reflected particles from the those for the case of a smooth surface, and the effect of roughness is particularly significant for ellipsoidal particles. At the same time, surface roughness reduces the influence of the particle shape, and the scattering indicatrices are close to each other for particles of all shapes.

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